Compromise Mixed Allocation in Multivariate Stratified Sampling Using Dynamic Programming Technique

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Abstract. The idea of "Mixed Allocation" in stratified sampling was introduced by [4]. The concept was further developed by several authors in different manner. In the present paper the authors worked out the "Compromise Mixed Allocation" for multivariate stratified sampling for more than one. Say "*p*" characteristics using Dynamic Programming Technique are defined on each population unit. It is assumed that the properties of the strata on which the grouping scheme of [4] is based are prevalent in the multivariate case also. Numerical examples are also presented to illustrate the computational details.

Keywords: Stratified sampling, optimum allocation, mixed allocation, multivariate stratified sampling, dynamic programming technique, compromise allocation, compromise mixed allocation, relative loss in efficiency.

1 Introduction

The literal meaning of the word "compromise" is "via media", but it has a special meaning in stratified sampling literature where different types of allocation procedure like Equal, proportional, optimum and several other allocations are present. Usually any one type of allocation is selected according to the nature of the population, the use of single allocation procedure is not advisable to all strata due to practical implications. In such situations, one can divide the strata into k different groups which are non-overlapping and exhaustive groups that are similar in nature internally. A particular type of allocation can then be applied to a particular group of strata depending on the nature of the group. [4] worked out the allocation using the above criterion and named it as "Mixed Allocation".

They formulated the problem of finding a mixed allocation as the following nonlinear programming problem (NLPP)

$$\text{Minimize} \qquad F(\alpha_j)=\sum_{j=1}^{k}\sum_{h\in I_j}\frac{W_h^2 S_h^2}{\alpha_j\beta_h} \tag{1.1}$$

$$\text{subject to} \qquad \sum_{j=1}^{k}\sum_{h\in I_j}\alpha_j c_h \beta_h \le C_0 \tag{1.2}$$

$$\text{and} \qquad \alpha_j \ge 0;\ j=1,2,\ldots,k \tag{1.3}$$

where L strata are divided into k groups, the j^{th} group consists of L_j strata. The sample allocations are given by

$$n_h=\alpha_j\beta_h; h\in I_j, j=1,2,\ldots,k \tag{1.4}$$

where α_j; $j=1,2,\ldots,k$ are the solution to NLPP (1.1) - (1.3), I_j is the set of integers representing the strata numbers in the j^{th} group and β_h is fixed according to the particular allocation used. For example

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if proportional allocation is to be used in the q^{th} group then $\beta_h = W_h$; $h \in I_q$.

In multivariate stratified sampling where p (> 1) characteristics are to be measured on each selected unit of the sample, an allocation that minimizes the variance of one characteristic may result in significant losses of precision for other characteristics. For such situations [26] gave the idea of "Compromise Allocation" which minimizes the weighted sum of variances of all the p-characteristics for a fixed budget this type of allocation is based on a compromise criterion to have a combined objective instead of several objectives (minimizing the individual variances). After that different authors suggested different compromise criteria or explored further the already existing criteria. Among them are [26], [6], [16], [20], [10], [11], [7], [2], [3], [24], [8], [9], [23], [17], [18], [19], [14], [15], [5] and many others. [21] discussed five different compromise criteria to work out approximate optimum allocation in multivariate surveys and compared them using a simulation study. [22] gave three different compromise criteria and modified the random search method to develop an algorithm to obtain the compromise allocation for multivariate stratified populations.

Before [4] no author used the term "Mixed Allocation" and thus no sampling literature is available on mixed allocation. However, [12] used a similar idea in univariate two-stage sampling design. In multivariate case instead of individual optimum allocations usually a compromise allocation is used.

[4] worked out the mixed allocation for univariate stratified sampling. In this paper we extended the work of [4] for the multivariate case using Dynamic Programming Technique (DPT). Thus the present paper presents a combination of Compromise and Mixed allocations. The allocation thus obtained, may be termed as "Compromise Mixed Allocation".

Section 2 gives the formulation and solution of the problem. Section 3 highlights the situation in which the compromise mixed allocation may be used in practice. Section 4 illustrates a numerical example to justify the use of compromise mixed allocation. Section 5 summarizes the comparative performance of the proposed allocation with some other compromise allocations. Section 6 gives the concluding remark on the basis of the results obtained in Sections 4 and 5.

A list of alphabetically arranged references is provided at the end of the paper.

2 Formulation

Using [26] criterion the problem of finding the mixed allocation given in (1.1) - (1.3) for multivariate case may be expressed as

$$\text{Minimize} \quad \sum_{l=1}^{p} a_l \sum_{j=1}^{k} \sum_{h \in I_j} \frac{W_h^2 S_{lh}^2}{\alpha_j \beta_h} \tag{2.1}$$

$$\text{subject to} \quad \sum_{j=1}^{k} \sum_{h \in I_j} \alpha_j \, c_{lh} \, \beta_h \le C_0 \tag{2.2}$$

$$\text{and} \quad \alpha_j \ge 0 \; ; \; j = 1,2,\ldots,k, \qquad C_0 = C - c_0 \tag{2.3}$$

where $a_l > 0$ is the weight assigned to the variance of the l^{th} characteristic, S_{lh}^2 is the stratum variance for the l^{th} characteristic, C is the total budget, c_0 is the overhead cost, C_0 is the available budget after deducting the overhead cost and $n_h = \alpha_j \beta_h$. Here-in-after c_h will denote the cost of measuring for all the p characteristics on a selected unit of h^{th} stratum, that is $c_h = \sum_{l=1}^{p} c_{lh}$ with $h = 1,2,\ldots,L$, and c_{lh} denoting the per unit cost of measurement for the l^{th} characteristic in the h^{th} stratum. Without loss of generality we can assume that $\sum_{l=1}^{p} a_l = 1$.

Substituting $A_h = W_h^2 \sum_{l=1}^{p} a_l S_{lh}^2$; $h = 1,2,\ldots,L$ and rearranging the terms NLPP (2.1) - (2.3) may be restated as

$$\text{Minimize} \quad \sum_{j=1}^{k} \sum_{h \in I_j} \frac{A_h}{\alpha_j \beta_h} \tag{2.4}$$

$$\textit{subject to} \quad \sum_{j=1}^{k} \sum_{h\in I_j} \alpha_j \, c_h \, \beta_h \le C_0 \tag{2.5}$$

$$\textit{and} \quad \alpha_j \ge 0; \; j = 1,2,\ldots,k \tag{2.6}$$

Since the objective function is convex and the constraint is linear at the optimal point the constraint will be active [1]. The NLPP (2.4) - (2.6) may be solved by Dynamic Programming Technique.

$$\left.\begin{aligned} &\textit{Minimize} && F(\alpha_j) = \sum_{j=1}^{k}\sum_{h\in I_j} \frac{A_h}{\alpha_j \beta_h} \\ &\textit{subject to} && \sum_{j=1}^{k}\sum_{h\in I_j} \alpha_j c_h \beta_h \le C_0 \\ &\textit{and} && \alpha_i \ge 0 \; ; \; j = 1,2,\ldots,k \end{aligned}\right\} \tag{2.7}$$

Consider the r^{th} stage sub problem of MPP (2.7) for the first r(<k) groups.

$$\left.\begin{aligned} &\textit{Minimize} && \sum_{j=1}^{r} f_j(\alpha_j) \\ &\textit{subject to} && \sum_{j=1}^{r} g_j(\alpha_j) \le C_r \\ &\textit{and} && \alpha_j \ge 0 \; ; \; j = 1,2,\ldots,r \end{aligned}\right\} \tag{2.8}$$

where $f_j(\alpha_j) = \sum_{h\in I_j}^{r} \frac{A_h}{\alpha_j \beta_h}$, $g_j(\alpha_j) = \sum_{h\in I_j}^{r} \alpha_j c_h \beta_h \le C_0$; $j = 1,2,\ldots,r$. $C_r < C_0$ is the available budget for measurements of the selected units from the first r groups. With the above definition of C_r we have

$C_r = C_0 \; for \; r=k$

also $C_r = g_1(\alpha_1) + g_2(\alpha_2) + \ldots + g_r(\alpha_r)$

$C_{r-1} = g_1(\alpha_1) + g_2(\alpha_2) + \ldots + g_{r-1}(\alpha_{r-1}) = C_r - g_r(\alpha_r)$

: :

: :

$C_2 = g_1(\alpha_1) + g_2(\alpha_2) = C_3 - g_3(\alpha_3)$

and $C_1 = g_1(\alpha_1) = C_2 - g_2(\alpha_2)$

If $f\left(r, C_r\right)$ denotes the minimum value of the objective function of sub problem (2.8), then

$$f\left(r, C_r\right) = \underset{feasible \; \alpha_j}{Min} \left\{ \sum_{j=1}^{r} f_j\left(\alpha_j\right) : \sum_{j=1}^{r} g_j\left(\alpha_j\right) = C_r \;\; and \; \alpha_j \ge 0 \; ; j = 1,2,\ldots,r \right\} \tag{2.9}$$

For the first stage (r=1)

$$f\left(1, C_1\right) = \frac{\left(\sum_{h\in I_1} \frac{A_h}{\beta_h}\right)\left(\sum_{h\in I_1} c_h \beta_h\right)}{C_1} \quad at \quad \alpha_1 = \frac{C_1}{\sum_{h\in I_1} c_h \beta_h} \tag{2.10}$$

and for r $\geq$ 2

$$f\left(r, C_r\right) = \min_{0 \le g_r(\alpha_r) \le C_r} \left\{ \frac{\sum_{h\in I_r} \frac{A_h}{\beta_h}}{\alpha_r} + f(r-1, C_r - g_r(\alpha_r)) \right\} \tag{2.11}$$

Expression (2.11) gives the required recurrence relation.

From $f\left(k,C\right)$ the optimum value of α_k is obtained, from $f\left(k-1, C-g_k(\alpha_k)\right)$ the optimum value of α_{k-1} is obtained and so on until α_1 is determined.

After obtaining $\alpha_j; j = 1,2,...,k$ the values of n_h are obtained by using (1.4).

$$V_{mixed} = \sum_{l=1}^{p} a_l \sum_{j=1}^{k} \sum_{h\in I_j} \frac{W_h^2 S_{lh}^2}{\alpha_j \beta_h} \tag{2.12}$$

3 Criterion for Using Compromise Mixed Allocation

The relative loss of efficiency (R. L. E.) by using different allocations in different groups of strata instead of optimum allocation is

$$R.\ L.\ E. = L(\underline{n}) = \frac{V_{mixed} - V_{opt}}{V_{opt}} \times 100\,\% \tag{3.1}$$

where $\underline{n} = (n_1, n_2, \ldots, n_L)$ is the vector of compromise mixed allocation.

$L(\underline{n})$ given by (3.1) will be the sum of the losses of the efficiencies incurred due to various allocations in different groups of strata. If any particular allocation results in a significant loss of efficiency, it may be replaced by any other more efficient allocation.

4 A Numerical Illustration

[4] gave a numerical illustration using artificial data. We added another characteristics to that data with the corresponding values of s_h as, s_{1h} and s_{2h}. Thus we have the following situation.

In stratification with seven strata and two characteristics the values of N_h, s_{1h}, s_{2h} and c_h are given in Table 1. It is assumed that the total available budget of the survey $C = 4500$ units, which includes an overhead cost $c_0 = 500$ units. This gives the total available amount for measurements

$$C_0 = C - c_0 = 4500 - 500 = 4000 \text{ units.}$$

Table 1. Data for seven strata and two characteristics

h	N_h	s_{1h}	s_{2h}	c_h	W_h
1	472	5.237	7.815	6	0.1888
2	559	5.821	7.949	8	0.2236
3	425	5.238	7.725	7	0.1700
4	218	25.528	30.125	12	0.0872
5	233	22.232	32.231	11	0.0932
6	328	15.129	18.455	10	0.1312
7	265	40.125	45.358	15	0.1060

The strata are so numbered that:

(i) Strata 1, 2 and 3 constitute group G_1 in which equal allocation is to be used, that is $\beta_h = 1;\ h\in I_1 = \{1,2,3\}$

(ii) Strata 4 and 5 constitute group G_2 in which proportional allocation is to be used, that is $\beta_h = W_h;\ h\in I_2 = \{4,5\}$

(iii) Strata 6 and 7 constitute group G_3 in which optimum allocation is to be used, that is $\beta_h = \sqrt{A_h/c_h};\ h\in I_3 = \{6,7\}$

Thus $I_1 = \{1,2,3\}$, $I_2 = \{4,5\}$ and $I_3 = \{6,7\}$.

It can be seen that $I_j; j = 1,2,3$ are mutually exclusive and exhaustive.

It is also assumed that both the characteristics are equally important that is $a_1 = a_2 = 0.5$.

Table 2. Values of A_h, A_h/β_h and $c_h\beta_h$

h	N_h	s_{1h}	s_{2h}	c_h	W_h	A_h	β_h	A_h/β_h	$c_h\beta_h$
1	472	5.237	7.815	6	0.1888	1.5773	1	1.5773	6
2	559	5.821	7.949	8	0.2236	2.4266	1	2.4266	8
3	425	5.238	7.725	7	0.1700	1.2588	1	1.2588	7
$h \in I_1$								5.2627	21
4	218	25.528	30.125	12	0.0872	5.92793	0.0872	67.981	1.046
5	233	22.232	32.231	11	0.0932	6.65843	0.0932	71.442	1.026
$h \in I_2$								139.423	2.072
6	328	15.129	18.455	10	0.1312	4.901	0.7001	7.0008	7.00
7	265	40.125	45.358	15	0.1060	20.600	1.1720	17.5796	17.58
$h \in I_3$								24.5804	24.58

$$\left.\begin{array}{ll} \textit{Minimize} & \dfrac{5.2627}{\alpha_1} + \dfrac{139.423}{\alpha_2} + \dfrac{24.5804}{\alpha_3} \\ \textit{subject to} & 21\alpha_1 + 2.072\alpha_2 + 24.58\alpha_3 \le 4000 \\ \textit{and} & \alpha_j \ge 0;\, j = 1,2,3 \end{array}\right\}$$

Using (2.10)

$$f(1, C_1) = \frac{21 \times 5.2627}{C_1} = \frac{110.5167}{C_1} \quad at \quad \alpha_1 = \frac{C_1}{21}$$

Using (2.11)

$$f(2, C_2) = \frac{139.423}{\alpha_2} + \frac{110.5167}{C_2 - 2.072\ \alpha_2}$$

Minimizing $f(2, C_2)$ with respect to α_2 by differentiating and equating to zero we get:

$$\frac{139.423}{\alpha_2^2} = \frac{110.5167 \times 2.072}{(C_2 - 2.072\ \alpha_2)^2}$$

This gives $\alpha_2 = 0.2982\, C_2$ and $f(2,\ C_2) = \dfrac{756.7612}{C_2}$

Using recurrence relation (2.11)

$$f(3, C_3) = \frac{24.5804}{\alpha_3} + \frac{756.7612}{C_3 - 24.58\ \alpha_3}$$

$$\Rightarrow \qquad \frac{24.5804}{\alpha_3^2} = \frac{756.7612 \times 24.58}{(C_3 - 24.58\ \alpha_3)^2}$$

or $\quad \alpha_3 = 0.0192\ C_3$

Thus $f(3,\ C_3) = \dfrac{2713.3153}{C_3}$

Putting $C_3 = C_0 = 4000$ we get

$$\alpha_3 = 0.0192\ C_3 = 76.8$$

Thus $C_2 = C_3 - 24.58\ \alpha_3 = 2112.256$
and $\alpha_2 = 0.2982\ C_2 = 629.8747$
Finally $C_1 = C_2 - 2.072\ \alpha_2 = 807.1556$
and $\alpha_1 = \dfrac{C_1}{21} = 38.4360$
Using (1.4) the values of n_h are obtained as

$$n_1 = \alpha_1\ \beta_1 = 38.4360 \times 1.00 \approx 38$$
$$n_2 = \alpha_1\ \beta_2 = 38.4360 \times 1.00 \approx 38$$
$$n_3 = \alpha_1\ \beta_3 = 38.4360 \times 1.00 \approx 38$$
$$n_4 = \alpha_2\ \beta_4 = 629.8747 \times 0.0872 = 54.9251 \approx 55$$
$$n_5 = \alpha_2\ \beta_5 = 629.8747 \times 0.0932 = 58.7043 \approx 59$$
$$n_6 = \alpha_3\ \beta_6 = 76.8 \times 0.7001 = 53.7677 \approx 54$$
$$n_7 = \alpha_3\ \beta_7 = 76.8 \times 1.172 = 90.0096 \approx 90$$

Table 3. Computation of α_j

Group No. j	(A) $\sum_{h\in I_j} A_h/\beta_h$	(B) $\sum_{h\in I_j} c_h\beta_h$	(C) $\sqrt{(A)/(B)}$	(D) $\sqrt{(A)(B)}$	$\alpha_j = 4000\dfrac{(C)}{\sum(D)}$
1	5.2627	21	0.5006	10.5127	38.4424
2	139.4226	2.0716	8.2038	16.9949	629.9918
3	24.5804	24.5810	1.0000	24.5807	76.7927

The computations are shown in Tables 2 and 3.

With the values of α_j; $j = 1,2,3$ given in last column of Table 3, the mixed allocation is obtained as:

For $j=1$ $\quad n_{1(m)} = n_{2(m)} = n_{3(m)} = \alpha_1 = 38.4424$

For $j=2$ $\quad n_{4(m)} = \alpha_2\beta_4 = \alpha_2 W_4 = 629.9918 \times 0.0872 = 54.9353$

$\quad n_{5(m)} = \alpha_2\beta_5 = \alpha_2 W_5 = 629.9918 \times 0.0932 = 58.7152$

For $j=3$ $\quad n_{6(m)} = \alpha_3\beta_6 = \alpha_3(\sqrt{A_6/c_6}) = 76.7927 \times 0.7001 = 53.7626$

$\quad n_{7(m)} = \alpha_3\beta_7 = \alpha_3(\sqrt{A_7/c_7}) = 76.7927 \times 1.1720 = 90.0010$

The estimated variance under mixed allocation given by (2.12) is $v_{mixed} = 0.6783$

Table 4 gives the sample sizes when overall optimum allocation is used. These values are required to work out R. L. E.

Table 4. Sample sizes under over all optimum allocation

h	W_h	A_h	c_h	$\sqrt{A_h/c_h}$	$\sqrt{A_h c_h}$	$n_{h(opt)}$
1	0.1888	1.5773	6	0.5127	3.0763	39.4205
2	0.2236	2.4266	8	0.5507	4.4060	42.3422
3	0.1700	1.2588	7	0.4241	2.9684	32.6082
4	0.0872	5.9279	12	0.7028	8.4341	54.0369
5	0.0932	6.6584	11	0.7780	8.5582	59.8189
6	0.1312	4.9013	10	0.7001	7.0009	53.8293
7	0.1060	20.6032	15	1.1720	17.5798	90.1128

The estimated variance under optimum allocation is given by

$$v_{opt} = \frac{\left(\sum_{h=1}^{L}\sqrt{A_h c_h}\right)^2}{C_0} = \frac{(52.0237)^2}{4000} = 0.6766 .$$

5 The Performance of Compromise Mixed Allocation as Compared to Some Other Allocation

In this Section a comparative study of the proposed compromise mixed allocation has been made with three other well known compromise allocations viz. Cochran's Average Allocation [13], Chatterjee's Compromise Allocation [10] and Compromise Allocation for "Minimizing Trace" [25]. However, these compromise allocations assume that the values of W_h and S_h^2 are known for all strata.

The Cochran's Average Compromise Allocation (ACA)

The individual optimum allocations n_{lh}^* are given by

$$n_{lh}^* = \frac{C_0 W_h S_{lh} \big/ \sqrt{c_h}}{\sum_{h=1}^{L} W_h S_{lh}\sqrt{c_h}}; \qquad l = 1, 2 .$$

The average compromise allocation $n_{h(ACA)}$ is given by

$$n_{h(ACA)} = \frac{1}{p}\sum_{l=1}^{p} n_{lh}^* ; \qquad h = 1, 2, \ldots, L .$$

Chatterjee's Compromise Allocation (CCA)

Chatterjee's compromise allocation $n_{h(CCA)}$, obtained by minimizing the sum of the relative increases in the variances of the estimates is given as

$$n_{h(CCA)} = \frac{C_0\sqrt{\sum_{l=1}^{p} n_{lh}^{*2}}}{\sum_{h=1}^{L} c_h \sqrt{\sum_{l=1}^{p} n_{lh}^{*2}}}; \quad h = 1, 2, \ldots, L .$$

Sukhatme's Compromise Allocation (SCA)

This compromise allocation $n_{h(SCA)}$, obtained by minimizing the trace of the variance-covariance matrix is given by

$$n_{h(SCA)} = \frac{C_0 W_h \sqrt{\sum_{l=1}^{p} S_{lh}^2 \big/ c_h}}{\sum_{h=1}^{L} W_h \sqrt{c_h \sum_{l=1}^{p} S_{lh}^2}}; \qquad h = 1, 2, \ldots, L .$$

These compromise allocations, for the data used in Section 4, are worked out and listed in Table 5.

Table 5. Various rounded off compromise allocations and the corresponding variances

h	Allocations	n_1	n_2	n_3	n_4	n_5	n_6	n_7	v_1	v_2	Trace= v_1+v_2	Cost incurred
1	Cochran's	39	42	32	54	59	54	91	0.52647	0.82826	1.35473	3996
2	Chatterjee's	39	42	32	54	59	54	91	0.52647	0.82826	1.35473	3996
3	Minimizing Trace	39	42	33	54	60	54	90	0.52671	0.82689	1.35360	3999
4	Proposed	38	38	38	55	59	54	90	0.52800	0.82963	1.35763	3997

Table 5 gives the rounded off values of the Cochran's, Chatterjee's, Sukhatme's and the Proposed compromise allocations, the variances v_1 and v_2 of the estimates of the two characteristics under study, the Trace (v_1+v_2) and the total cost incurred. It can be seen that the proposed allocation is almost as precise as the other allocations (in terms of the 'Trace') that assume the knowledge of the true values of W_h and S_h^2 for all strata. Whereas the proposed allocation may be used in this conditions is given in this manuscript.

6 Conclusion

Since the estimated relative loss in efficiency of the compromise mixed allocation as compared to the overall optimum allocation is

$$(\text{R. L. E.})_{mixed} = \frac{v_{mixed} - v_{opt}}{v_{opt}} \times 100\% = \frac{0.6783 - 0.6766}{0.6766} \times 100\% = 0.2513\%$$

which is very small, the proposed compromise mixed allocation may be used without any significant loss in the efficiency. In addition to the above fact the compromise mixed allocation also works well in comparison with other compromise allocations discussed in Section 5.

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