Advances in Astrophysics, Vol. 1, No. 2, August 2016
https://dx.doi.org/10.22606/adap.2016.12006

Locally Rotationally Symmetric Bianchi Type I Massive String Cosmological Models with Bulk Viscosity and Decaying Vacuum Energy Density

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Abstract. We examine locally rotationally symmetric Bianchi Type I massive string cosmological models with bulk viscosity and time varying cosmological term (Λ) (vacuum energy density). To get the deterministic models of the universe, we consider two cases: (i) $\sigma \ \alpha \ \theta$, $\zeta\theta =$ constant, $\Lambda \sim \frac{1}{R^2}$; (ii) $\rho = 3H^2$, $\zeta \ \alpha \ \rho^{1/2}$, $\Lambda = 3\beta H^2$ where σ is the shear, θ the expansion, ζ the coefficient of bulk viscosity, ρ the energy density, Λ the vacuum energy density and H the Hubble parameter. Both models satisfy energy conditions and represent anisotropic space-time. The first model represents accelerating behaviour of the universe while the second model represents accelerating and decelerating phases of the universe. The first model starts with a big-bang at $T = 0$ and the expansion decreases with time. While the second model also starts with big-bang at $\tau = 0$ and the expansion vanishes for large value of τ. For both models $\Lambda \sim \frac{1}{T^2}$ and $\Lambda = \frac{1}{\tau^2}$, these results match the result obtained by Beesham [27]. Both models have Point Type singularities at $T = 0$ and $\tau = 0$ respectively.

Keywords: Bianchi I, massive string, cosmological, bulk viscous, vacuum energy density.

1 Introduction

The presence of strings in the early universe can be explained using grand unified field theories as investigated by Kibble [1,2], Everett [3] and Vilenkin [4]. It is believed that the very early universe underwent phase transition giving some topological stable structure. Strings have interesting cosmological consequences among the various topological defects that occurred during the phase transition and before the creation of particles in the early universe (Kibble [1]). Zel'dovich [5] in his investigation has pointed out that cosmic strings cause density perturbations leading to the formation of galaxies. These strings have stress energy and couple with gravitational field. The pioneering work in the formulation of energy-momentum tensor for classical massive strings was initiated by Letelier [6,7] who explained that massive strings are formed by geometric string with particles attached along its extension and used this idea to find string cosmological solutions using Bianchi Type I and Kantowski-Sachs space-times. As Einstein's theory of gravitation is the theory for understanding the nature and evolution of the large scale structure of the universe, it is, therefore, interesting to study the gravitational effects which arise from strings within the frame work of Einstein gravity. In literature, string cosmological models for different Bianchi Types (homogeneous and anisotropic) space-times have been widely studied in different physical and geometrical contexts by many authors viz. Banerjee et al. [8], Tikekar and Patel [9,10], Ilhami and Tarhan [11], Bali et al. [12-18], Wang [19], Reddy et al. [20], Rao et al. [21,22], Mahanto et al [23].

The introduction of viscosity in the cosmic fluid content has been very useful in explaining many physical aspects of dynamics of homogeneous cosmological models. The dissipation mechanism not only modifies the nature of singularity but also successfully explains the large entropy per baryon in the present universe. There are several processes which give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during recombination era. Bulk viscosity is also associated with grand unified theory phase transition and string creation and can lead to the inflationary universe scenario. It is well known that in the early stage of the universe when neutrino decoupling occurred, the matter behaved like viscous fluid (Klimek [24]). The coefficient of viscosity is known to decrease as the universe expands. The role of viscosity in cosmology has been studied by several authors viz. Pimentel [25], Berman [26], Beesham [27], Arbab [28], Pavon et al. [29], Zimdahl [30], Gron[31], Saha [32,33], Singh et al. [34], Bali et al. [35,36], Mostafapoor and Gron [37].

In modern cosmological theories, the cosmological term $\Lambda(t)$ is a focal point of interest as it solves the cosmological constant problem in a natural way. The current accelerating expansion of the universe suggests that our universe is dominated by unknown dark energy. The cosmological constant is the most favoured candidate of dark energy representing energy density of vacuum in the context of quantum field theory. The cosmological constant occupies a privileged place in the dark energy models because it provides a good approximation to the present astronomical data as studied by Zel'dovich [38], Dreitlein [39], Krauss and Turner [40]. The observations for distant type Ia supernovae (Perlmutter et al. [41,42], Riess et al. [43,44], Garnavich et al. [45,46], Schmidt et al. [47]) strongly favour a positive value of Λ in order to measure the expansion rate of universe and now it is believed that the universe is not only expanding but also accelerating. Many authors viz. Bertolami [48], Chen and Wu [49], Sahni and Starobinsky [50], Verma and Ram [51], Pradhan and Singh [52], Bali et al. [53,54] investigated cosmological models with decaying vacuum energy density (Λ). Recently Bali and Singh [55] investigated LRS Bianchi Type II massive string cosmological model for stiff fluid distribution with decaying vacuum energy (Λ).

We have investigated some massive string cosmological models with bulk viscosity and vacuum energy density in LRS Bianchi Type I space-time. To get the deterministic models of the universe, we consider two cases: (i) $\sigma \ \alpha \ \theta$, $\zeta\theta$ = constant and $\Lambda \sim \frac{1}{R^2}$; (ii) $\rho = 3H^2$, $\zeta \ \alpha \ \rho^{1/2}\Lambda = 3\beta H^2$ where σ is shear, θ the expansion, ζ the coefficient of bulk viscosity, Λ the vacuum energy density, R the scale factor, ρ the energy density, and H the Hubble parameter. In the first case, we find that the model satisfies strong and weak energy conditions and the model represents accelerating behaviour of the universe and is anisotropic. In the second case, the model also satisfies energy conditions. The second model represents accelerating and anisotropic space-time. The vacuum energy density $\Lambda \sim \frac{1}{\tau^2}$ as obtained by Beesham [27]. We have also calculated state finder parameters {r,s} and these lead to {1,0} in special case.

2 Metric and Field Equations

For simplification and large scale behaviour of actual universe, LRS Bianchi Type I models have great importance. Lidsey [56] in his investigation has pointed out these models are equivalent to FRW models which are considered standard models of our universe.

We consider locally rotationally symmetric Bianchi Type I space-time in the form as

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \tag{1}$$

where A and B are metric potentials and are functions of t-alone. The energy-momentum tensor for a cloud of string with bulk viscosity is given by Letelier [7] and Landau and Lifshitz [57] as

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \zeta\theta \ (g_i^j + v_i v^j) \tag{2}$$

with $v_i v^i = -x_i x^i = -1$, and

$$x_1 \neq 0, \ x_2 = 0 = x_3 = x_4 \tag{3}$$

where ρ is the proper energy density with particles attached to them, λ the string tension density, v^i the four velocity of particles, x^i the unit space-like vector representing the direction of string, ζ the coefficient of bulk viscosity and θ the expansion in the model. If the particle density of configuration is denoted by ρ_p then for massive string, we have

$$\rho = \rho_p + \lambda \tag{4}$$

In comoving coordinate system, we have

$$v^i = \left(0,0,0,\frac{1}{A}\right) \text{ and } x^i = \left(\frac{1}{A},0,0,0\right) \tag{5}$$

The Einstein field equations

$$R_i^j - \frac{1}{2} R\, g_i^j = -T_i^j + \Lambda g_i^j \tag{6}$$

for the metric (1) lead to

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = \Lambda + \lambda + \zeta\theta \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = \Lambda + \zeta\theta \tag{8}$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} = \rho + \Lambda \tag{9}$$

3 Solution of Field Equations

To get the deterministic models of universe, we consider two cases:

(i) $\sigma\, \alpha\, \theta, \zeta\theta = \text{constant}$, $\Lambda = \frac{\alpha}{R^2}$ (Chen and Wu [49])

(ii) $\rho = 3H^2$, $\zeta\, \alpha\, \rho^{1/2}$, $\Lambda = 3\beta H^2$ (Barrow [58])

Now we consider **case (i)**: To get the deterministic solution in terms of cosmic time t, we assume that σ (shear) is proportional to expansion (θ).
Thus, we have

$$A = B^n \tag{10}$$

where A and B are metric potentials and n is a constant. The motive for assuming the condition $\sigma\ \alpha\ \theta$ is explained as: Referring to Thorne [59], the observations of velocity-red shift relation for extra galactic sources suggest that the Hubble expansion of universe is isotropic within 30% (Kantowski and Sachs [60], Kristian and Sachs [61]) and red-shift studies place the limit $\frac{\sigma}{H} \le 0.30$. Also Collins et al. [62] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous hyper-surface satisfies the condition $\frac{\sigma}{\theta} =$ constant.

It is well known that coefficient of bulk viscosity is known to decrease as universe expands. Thus, we consider $\zeta\theta$ = constant as given by Zimdahl [30]. We assume the condition $\Lambda = \frac{\alpha}{R^2}$ as considered by Chen and Wu [49] where R is scale factor.

Using the condition (10) $\zeta\theta$ = constant = k and $\Lambda = \frac{\alpha}{R^2}$ in equation (8), we have

$$2B_{44} + \frac{2n^2}{n+1}\frac{1}{B}B_4^2 = \frac{2}{n+1}\left[kB + \alpha B^{-\frac{(2n+1)}{3}}\right] \tag{11}$$

where

$$R^3 = AB^2 = B^{n+2} \tag{12}$$

and

$$\frac{\alpha}{R^2} = \frac{\alpha}{B^{\frac{2(n+2)}{3}}} \tag{13}$$

To get the solution of (11), we assume

$$B_4 = f(B)$$

which leads to

$$B_{44} = f f', \quad f' = df/dB$$

Thus equation (11) leads to

$$\frac{d}{dB}(f^2) + \frac{2n^2}{n+1}\frac{1}{B}f^2 = \frac{2}{n+1}\left[kB + \alpha B^{-\left(\frac{2n+1}{3}\right)}\right] \tag{14}$$

which leads to

$$f^2 = \frac{k}{n^2+n+1}B^2 + \frac{6\alpha}{4n^2+2}B^{\frac{2-2n^2}{3(n+1)}} \tag{15}$$

To find the solution in terms of cosmic time t, we assume n = 2. Thus equation (15) leads to

$$\left(\frac{dB}{dt}\right)^2 = f^2 = \frac{k}{7}B^2 + \frac{\alpha}{3}B^{-2/3} \tag{16}$$

Thus, we have

$$B^{4/3} = a \sinh(bt + \ell) \tag{17}$$

$$A = B^2 = a^{3/2}\sinh^{3/2}(bt + \ell) \tag{18}$$

where $a = \sqrt{\frac{7\alpha}{3k}}$, $b = \frac{4}{3}\sqrt{k/7}$

After suitable transformation of coordinates, the metric (1) leads to

$$ds^2 = -dT^2 + a^3\sinh^3 bT\, dX^2 + a^{3/2}\sinh^{3/2} bT(dY^2 + dZ^2) \tag{19}$$

where $bt + \ell = bT$, $x = X$, $y = Y$, $z = Z$.

In the absence of bulk viscosity, the metric (19) leads to

$$ds^2 = -dT^2 + T^3dX^2 + T^{3/2}(dY^2 + dZ^2) \tag{20}$$

Case (ii): We have

$$\rho = 3H^2, \zeta = \alpha\rho^{1/2}, \theta = 3H \tag{21}$$

as considered by Barrow [58] and $\Lambda = 3bH^2$ as considered by Arbab [28]

The conservation equation

$$(T_i^j + \Lambda g_i^j)_{;j} = 0 \tag{22}$$

leads to

$$\dot{\rho} + (\rho - \zeta\theta)\theta + \dot{\Lambda} = 0 \tag{23}$$

where $\theta = A_4/A + 2B_4/B$.

Using (21) and (22) in (23), we have

$$6H\dot{H} + (3H^2 - 3\sqrt{3}\alpha(3H) + 6\beta H\dot{H} = 0$$

which leads to

$$(6 + 6\beta)H\dot{H} + (9 - 9\sqrt{3}\alpha)H^3 = 0$$

Thus, we have

$$-\frac{\dot{H}}{H^2} = \frac{(3 - 3\sqrt{3}\alpha)}{2 + 2\beta} = \ell_2 > 0 \tag{24}$$

Equation (24) leads to

$$H = \frac{1}{\ell_2 t + \ell_3} \tag{25}$$

where ℓ_3 is constant of integration.

We have

$$\theta = \frac{A_4}{A} + \frac{2B_4}{B}$$

Thus

$$\theta = \frac{4B_4}{B} \text{ as } A = B^2 \tag{26}$$

Also

$$\theta = 3H$$

This leads to

$$\frac{4B_4}{B} = \frac{3}{\ell_2 t + \ell_3} \tag{27}$$

Equation (27) leads to

$$\frac{B_4}{B} = \frac{3}{4(\ell_2 t + \ell_3)} \tag{28}$$

We have

$$B = (\ell_2 t + \ell_3)^{3/2\ell_2}$$

and

$$A = B^2 = (\ell_2 t + \ell_3)^{3/2\ell_2} \tag{29}$$

After suitable transformation of coordinates, the metric (1) leads to

$$ds^2 = -d\tau^2 + \tau^{3/\ell_2} dX^2 + \tau^{3/2\ell_2}(dY^2 + dZ^2) \tag{30}$$

where $\ell_2 t + \ell_3 = \ell_2 \tau$, $x = X$, $y = Y$, $z = Z$.

4 Some Physical and Geometrical Features

The energy density (ρ), total energy density (ρ+Λ), vacuum energy density (Λ), the string tension density (λ), the particle density (ρ_p), the spatial volume (R^3), the shear (σ), the expansion (θ), the Hubble parameter (H), the deceleration parameter (q) for the model (19) are given by

$$\rho = \left(\frac{45b^2}{16} - \frac{\alpha}{a^2}\right)\coth^2 T + \frac{\alpha}{a^2} \tag{31}$$

$$\rho + \Lambda = \frac{45b^2}{16}\coth^2 T \tag{32}$$

$$\Lambda = \frac{\alpha}{a^2}(\coth^2 T - 1) \tag{33}$$

$$\lambda = \left(\frac{3b^2}{16} - \frac{\alpha}{a^2}\right)\coth^2 T + \frac{3b^2}{2} + \frac{\alpha}{a^2} - k \tag{34}$$

$$\rho_p = \frac{21b^2}{8}\coth^2 T - \frac{3b^2}{2} + k \tag{35}$$

$$R^3 = a^3 \sinh^3 T \tag{36}$$

$$\sigma = \frac{1}{\sqrt{3}}\left|\frac{A_4}{A} - \frac{B_4}{B}\right| = \frac{3b}{4}\coth T \tag{37}$$

$$\theta = 3b \coth T = \left(\frac{A_4}{A} + \frac{2B_4}{B}\right) = \frac{4B_4}{B} = 3b \coth T \tag{38}$$

$$\frac{\sigma}{\theta} = \frac{1}{4} \neq 0 \tag{39}$$

$$H = b \coth T \tag{40}$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\tan h^2 T \tag{41}$$

The above mentioned quantities for the model (30) are given by

$$\rho = \left(\frac{45}{16} - 3\beta\right)\frac{1}{\tau^2} \tag{42}$$

$$\rho + \Lambda = \frac{45}{16}\frac{1}{\tau^2} \tag{43}$$

$$\Lambda = \frac{3\beta}{\tau^2} \tag{44}$$

$$\lambda = \left(\frac{27}{16} - \frac{3\ell_2}{2} - 3\sqrt{3}\alpha - 3\beta\right)\frac{1}{\tau^2} \tag{45}$$

$$\rho_p = \rho - \lambda = \left(\frac{9}{8} + \frac{3\ell_2}{2} + 3\sqrt{3}\alpha\right)\frac{1}{\tau^2} \tag{46}$$

$$R^3 = AB^2 = \tau^{3/\ell_2} \tag{47}$$

$$\sigma = \frac{1}{\sqrt{3}}\left|\frac{A_4}{A} - \frac{B_4}{B}\right| = \frac{\sqrt{3}}{4}\frac{1}{\tau} \tag{48}$$

$$\theta = \frac{3}{\tau} \tag{49}$$

$$\frac{\sigma}{\theta} = \frac{1}{4\sqrt{3}} \tag{50}$$

$$H = \frac{1}{\tau} \tag{51}$$

$$q = -1 - \frac{\dot{H}}{H^2} = -1 + \ell_2 \begin{cases} < 0 \text{ if } \ell_2 < 1 \\ > 0 \text{ if } \ell_2 > 1 \end{cases} \tag{52}$$

ℓ_2 being constant of integration.

5 State Finder Parameters {r,s}

The state finder parameters effectively differentiate between forms of dark energy and provide simple diagnosis on whether a particular model fits into the basic observational data. Following Sahni et al. [63], the state finder diagnostic pair {r,s} is given by

$$r = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} = 1 + \frac{3\ell_2}{\tau^2} \tag{53}$$

$$s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} = \frac{3\ell_2 / \tau^2}{3\left(\ell_2 - \frac{3}{2}\right)} \tag{54}$$

where ℓ_2 is constant of integration. However, if $\ell_2 = 0$ then $r = 1$, $s = 0$ which agrees with Λ CDM model.

6 Conclusion

The weak energy condition (i) $\rho \geq \lambda$, $\lambda \geq 0$ as well as strong energy condition (ii) $\rho \geq 0$, $\lambda < 0$ given by Hawking and Ellis [64] for the model (19) are satisfied if

(i) $\coth^2 T \geq \dfrac{4(3b^2 - 2k)}{21b^2}$ and $\dfrac{3b^2}{16} \geq \dfrac{\alpha}{a^2}, \dfrac{3b^2}{2} + \dfrac{\alpha}{a^2} \geq k$

(ii) $\dfrac{45b^2}{16} \geq \dfrac{\alpha}{a^2}$, $\left(\dfrac{3b^2}{16} - \dfrac{\alpha}{a^2}\right)\coth^2 T < k - \dfrac{3b^2}{2} - \dfrac{\alpha}{a^2}$

Also for the model (30) these conditions lead to

(i) $\dfrac{9}{8} + 3\sqrt{3}\alpha \geq \dfrac{3\ell_2}{2}$ and $\dfrac{27}{16} \geq \dfrac{3\ell_2}{2} + \dfrac{3\sqrt{3}}{\alpha} + 3\beta$

(ii) $\dfrac{15}{16} \geq \beta, \dfrac{27}{16} < \dfrac{3\ell_2}{2} + 3\sqrt{3}\alpha + 3\beta$

For the model (19), energy density (ρ), the string tension (λ), the particle density (ρ_p) are initially large but for large value of T, these tend to finite quantity. The spatially volume increases exponentially representing inflationary scenario. Since $\dfrac{\sigma}{\theta} \neq 0$, hence the model is anisotropic space-time throughout. The model starts with a big-bang at T = 0 and the expansion decreases with time. $\Lambda \sim \dfrac{1}{T^2}$ which matches the result as obtained by Beesham [27]. The decelerating parameter q<0 which shows that the model represents accelerating universe. The model has Point Type singularity at T = 0 (MacCallum [65]). In the absence of bulk viscosity, the model is well defined and energy conditions are satisfied.

Also for the model (30), the energy density (ρ), the string tension (λ), the particle density (ρ_p) are initially large but tend to zero when $\tau \to \infty$. The spatial volume increases with time. Since $\dfrac{\sigma}{\theta} \neq 0$, hence the model represents anisotropic space-time, but at late time, it isotropizes. This result matches recent astronomical observations. The model starts with a big-bang at $\tau = 0$ and the expansion decreases with time. The model represents accelerating and decelerating phases of universe if $\ell_2 < 1$ and $\ell_2 > 1$ respectively. The viscosity prevents the matter density to vanish for the model (30). Also the model has Point Type singularity at $\tau = 0$. The vacuum energy density $\Lambda = \dfrac{3\beta}{\tau^2}$ which matches the result as obtained by Beesham [27]. We have also calculated state finder parameters {r,s} for the model (30). These parameters agree with Λ CDM model in special case.

Acknowledgments. The authors are thankful to the Referee for valuable comments and suggestions.

References

1. T.W.B. Kibble, "Topology of cosmic domains and strings", J. Phys. A, vol. 9, pp. 1387-1398, 1976.
2. T.W.B. Kibble, "Some implications of a cosmological phase transition", Phys. Reports, vol. 67, pp. 183-199, 1980.
3. A.E. Everett, "Cosmic strings in unified gauge theories", Phys. Rev. D, vol. 24, pp. 858-868, 1981.
4. A. Vilenkin, "Cosmic strings", Phys. Rev. D, vol. 24, pp. 2082-2089, 1982.
5. Ya. B. Zel'dovich, "Cosmological perturbations produced near a singularity", Mon. Not. Roy. Astron. Soc., vol. 192, pp. 663-667, 1980.
6. P.S. Letelier, "Clouds of strings in general relativity", Phys. Rev. D, vol. 20, pp. 1294-1302, 1979.
7. P.S. Letelier, "String Cosmologies", Phys. Rev. D, vol. 28, pp. 2414-2419, 1983.
8. A. Banerjee, A.K. Sanyal and S. Chakravorty, "String cosmology in Bianchi Type I space-time", Pramana – J.Phys., vol. 34, pp. 1-11, 1990.
9. R. Tikekar and L.K. Patel, "Some exact solution of string cosmology in Bianchi type III space time", Gen. Relativ. Gravit., vol. 24, pp. 397-404, 1996.
10. R. Tikekar and L.K. Patel, "Some exact solution in Bianchi type VI0 string cosmology", Pramana-J. Phys., vol. 42, pp. 483-489, 1994.

11. Y. Ilhami and I. Tarhan, "Some string cosmological models in Bianchi type I space time", Astrophys. and Space-Sci., vol. 240, pp. 45-54, 1996.
12. R. Bali and S. Dave, "Bianchi type III string cosmological model with bulk viscous fluid in general relativity", Astrophys. and Space-Sci., vol. 282, pp. 461-466, 2002.
13. R. Bali and R.D. Upadhaya, "An LRS Bianchi type I bulk viscous fluid string cosmological model in general relativity", Astrohys. and Space-Sci., vol. 288, pp. 287-292, 2003.
14. R. Bali and Anjali, "Bianchi type I magnetized string cosmological model in general relativity", Astrophys. and Space-Sci., vol. 302, pp. 201-205, 2006.
15. R. Bali and A. Pradhan, "Bianchi type III string cosmological models with time dependent bulk viscosity", Chin. Phys. Lett., vol. 24, pp. 585-588, 2007.
16. R. Bali and S. Jain, "Bianchi type V magnetized string dust cosmological models in general relativity", Int. J. Mod. Phys. D, vol. 16, pp. 11-20, 2007.
17. R. Bali, "Bianchi type V magnetized string dust bulk viscous fluid cosmological model with variable magnetic permeability", Int. J. Theor. Phys., vol. 48, pp. 476-486, 2009.
18. R. Bali and S. Singh, "Locally rotationally symmetric Bianchi type II massive string cosmological model for barotropic fluid distribution and vacuum energy density", J. of Cosmology, vol. 24, pp. 1-10, 2014.
19. X.X. Wang, "Exact solution for string cosmology", Chin. Phys. Lett., vol. 20, pp. 615-617, 2003.
20. D.R.K. Reddy, R.L. Naidu and U.V.M. Rao, "A cosmological model with negative constant deceleration parameter in Brans-Dicke theory", Int. J. Theor. Phys., vol. 46, pp. 1443-1448, 2007.
21. V.U.M. Rao, T. Vinutha and M.V. Shanthi, "Bianchi type V cosmological model with perfect fluid using negative constant deceleration parameter in a scalar theory based on Lyra manifold", Astrophys. and Space-Sci., vol. 314, pp. 213-216, 2008.
22. V.U.M. Rao and K.V.S. Sireesha, "Axially symmetric string cosmological model with bulk viscosity in self creation theory of gravitation", Euro. Phys. J. Plus, vol. 127, p. 49, 2012.
23. K.L. Mahanto, S.K. Biswal, P.K.Sahoo and M.C.Adhikary, "String cloud with quark matter in self creation cosmology", Int. J. Theor. Phys., vol. 51, pp. 1538-1544, 2012.
24. Z. Klimek, "Entropy per particle in the early Bianchi type I universe", Nuovo. Cimento B, vol. 35, pp. 249-258, 1976.
25. L.O. Pimentel, "Exact self creation cosmological solutions", Astrophys. Space-Sci., vol. 116, pp. 395-399, 1985.
26. M.S. Berman, "Cosmological models with a variable cosmological term", Phys. Rev. D, vol. 43, pp. 1075-1078, 1990.
27. A. Beesham, "Cosmological models with a variable cosmological term and bulk viscous models", Phys. Rev. D, vol. 48, pp. 3539-3543, 1993.
28. A.I. Arbab, "Cosmological models with variable cosmological and gravitational constants and bulk viscous models, Gen. Relativ. Gravit., vol. 29, pp. 61-74, 1997.
29. D. Pavon, J. Bafaluy and D. Jou, "Causal Friedmann-Robertson-Walker cosmology", Class. Quant. Gravity, vol. 8, pp. 347-360, 1991.
30. W. Zimdahl, "Bulk viscous cosmology", Phys. Rev. D, vol. 53, pp. 5483-5493, 1996.
31. Gron, O. "Viscous inflationary universe models", Astrophys. and Space Science, vol. 173, pp. 191-225, 1990.
32. B. Saha, "Bianchi type I universe with viscous fluid", Mod. Phys. Lett. A, vol. 20, pp. 2127-2143, 2005.
33. B. Saha, "Anisotropic cosmological models with spinor field and viscous fluid in presence of Lambda term", J. Phys. A: Math. Theor., vol. 40, pp. 14011-14027, 2007.31. Gron, O. "Viscous inflationary universe models", Astrophys. and Space Science, vol. 173, pp. 191-225, 1990.
34. C.P. Singh, S. Kumar and A. Pradhan, "Early viscous universe with variable gravitational and cosmological constants", Class. Quant. Gravity, vol. 24, pp. 455-474, 2007.
35. R. Bali and J.P. Singh, "Bulk viscous Bianchi type I cosmological models with time dependent cosmological term lambda", Int. J. Theor. Phys., vol. 47, pp. 3288-3297, 2008.
36. R. Bali, P. Singh and J.P. Singh, "Bianchi type V viscous fluid of cosmological models in presence of decaying vacuum energy", Astrophys. and Space. Sci., vol. 341, pp. 701-706, 2012.
37. N. Mostafapoor and O. Gron, "Viscous CDM universe models", Astrophys. and Space Sci., vol. 333, pp. 357-368, 2011.

38. Ya. B. Zel'dovich, "The cosmological constant and theory of elementary particles", Sov. Phys. Uspekhi., vol. 11, pp. 381-393, 1968.
39. J. Dreitlein, "Broken symmetry and the cosmological constant", Phys. Rev. Lett., vol. 33, pp. 1243-1244, 1974.
40. L.M. Krauss and M.S. Turner, "The cosmological constant is back", Gen. Relativ. Gravit., vol. 27, pp. 1137-1144, 1995.
41. S. Perlmutter et al., "Discovery of supernova explosion at half the age of universe", Nature, vol. 391, pp. 51-54, 1998.
42. S. Perlmutter et al., "Measurements of and from 42 high redshift supernovae", Astrophys. J., vol. 517, pp. 565-586, 1999.
43. A.G. Riess et al., "Observational evidence from supernovae", Astron. J., vol. 116, pp. 1009-1038, 1998.
44. A.G. Riess et al., "Type Ia supernova discoveries at 271 from Hubble space telescope", Astropohys. J., vol. 607, pp. 665-687, 2004.
45. P.M. Garnavich et al., "Constraints on cosmological models from Hubble space Telescope", Astrophys. J., vol. 493, pp. L 53 - L 57, 1998.
46. P.M. Garnavich et al., "Super nova limits on the cosmic equation of state", Astrophys. J., vol. 509, pp. 74-79, 1998.
47. B.P. Schmidt et al., "The High Z-supernova search evidence for past deceleration and constraints on dark energy evolution", Astrophys. J., vol. 507, pp. 46-63, 1998.
48. O. Betrolami, "Time dependent cosmological term", Nuovo Cim. B, vol. 93, p. 36, 1986.
49. W. Chen and Y.S. Wu, "Implication of cosmological constant varying as R-2", Phys. Rev., vol. 041, pp. 695-698, 1990.
50. V. Sahni and A. Starobinsky, "The case for a positive cosmological -term, Int. J. Mod. Phys. D, vol. 9, pp. 373-443, 2000.
51. M.K. Verma and S. Ram, "Bulk viscous Bianchi type III model with time dependent G and , Int. J. Theor. Phys., vol. 49, pp. 693-700 (2009).
52. A. Pradhan and S.K. Singh, Bianchi Type I magneto fluid cosmological models with variable cosmological constants - revisited, Int. J. Mod. Phys. D, vol. 13, pp. 503-516, 2004.
53. R. Bali, P. Singh and J.P. Singh, "Viscous Bianchi type I universe with stiff matter and decaying vacuum energy density", ISRN Mathematical Phys., vol. 2012 article ID 704612, pp. 1-11, 2012.
54. R. Bali and S. Singh, "LRS Bianchi type II inflationary universe with massless scalar filed and time varying ", Chin. Phys. Lett., vol. 29, p. 080404, 2012.
55. R. Bali and S. Singh, "LRS Bianchi type II massive string cosmological model for stiff fluid distribution with decaying vacuum energy", Int. J. Theor. Phys., vol. 53, pp. 2082-2090, 2014.
56. J.E. Lidsey, "The scalar field as dynamical variable in inflation", Phys. Lett. B, vol. 273, pp. 42-46, 1991.
57. L.D. Landau and E.M. Lifshitz, Fluid Mechanics, Pergamon Press, London 1976.
58. J.D. Barrow, "Cosmic no-hair theorem and inflation", Phys. Lett. B, vol. 187, pp. 12-16, 1987.
59. K.S.Thorne, "Primordial element formation, primordial magnetic fields and the isotropy of the universe", Astrophys. J., vol. 148, pp. 51-68, 1967.
60. R. Kantowski and R.K Sachs, "Some spatially homogenous anisotropic relativistic cosmological models", J. Math. Phys., vol. 7, pp. 443-446, 1966.
61. J. Kristian and R.K. Sachs, "Observation in cosmology", Astrophys., J., vol. 143, pp. 379-399, 1966.
62. C.B. Collins, E.N. Glass and D.A. Wilkinson, "Exact spatially homogenous cosmologies", Gen. Relativ. Gravit., vol. 12, pp. 805-823, 1980.
63. V. Sahni, T.D. Saini, A. Starobinsky and U. Alam, "State finder a new geometrical diagnostic of dark energy", JETP Lett., vol. 77, pp. 201-206, 2003.
64. S.W. Hawking and G.F.R. Ellis, The large scale structure of space-time, Cambridge Univ. Press, England, p.88, 1974.
65. M.A.H. MacCallum, "A class of homogeneous cosmological models III: asymptotic behaviour", Comm. Math. Phys., vol. 20, pp. 57-84, 1971.