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Interior Resonance Periodic Orbits in Photogravitational Restricted Three-body Problem

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Abstract. Interior resonance periodic orbits around the Sun in the Sun-Jupiter photogravitational restricted three-body problem using the method of Poincaré surface of section are studied. The nature, stability and location of these periodic orbits with interior resonances of 2:1, 3:2 and 4:3 are generated for different values of Jacobi constant C. It is found that with the increase in the value of C, these orbits transform to tidal lock, which is a rare case of resonance with 1:1. The period of time for these orbits is found to decrease with the increase in the Sun's radiation pressure. The oblateness of Jupiter is found to increase the period of time for these orbits marginally.

Keywords: Interior resonance, periodic orbits, Poincaré surface of section, solar radiation pressure, oblateness, tidal lock.

1 Introduction

The importance of the phenomena of resonance in the dynamic evolution of the solar system was studied by Roy and Ovenden [1]. They established that among the planetary and satellite systems, the occurrence of commensurability between the pairs of mean motions is more frequent than in a chance distribution. The existence of a mean motion resonance between a pair of objects can lead to a repeating geometrical configuration of the orbits which guarantees stability even if the resonance is not exact, since there is still the possibility of stable liberal motion around an equilibrium point. Therefore, it is important to have an understanding of the dynamics of resonance and to develop analytical models that precisely reflect the true nature of resonant interactions. Since the late twentieth century until today, the enormous number of researches have enriched the study of Restricted Three-Body Problem (RTBP), but the influence of the various perturbing forces has not been studied in many of such interesting problems. Two such forces are due to radiation pressure and oblateness. We wish to study the effect of these perturbing forces on the interior resonance periodic orbits.

The radiation force on a particle exerted by a radiating body generally consists of three terms, namely the radiation pressure, the Doppler shift of the incident radiation and the Poynting drag (Poynting [2]; Robertson [3]). The first two are radial and the third one acts opposite to the velocity vector. The latter two components are caused by absorption and subsequent re-emission of radiation and constitute Poynting-Robertson effect. Radzievskii [4] pointed out that these two effects are negligible and that the only significant force is due to radiation pressure. A number of researchers, including Chernikov [5], Perezhogin [6], Bhatnagar and Chawla [7], Schuerman [8], Simmons et al. [9], Roman [10], Kushvah and Ishwar [11] and Das et al. [12] carried out further studies to understand the motion of objects under the effect of solar radiation pressure in the RTBP with radiating primaries. The restricted problem when the three participating bodies are oblate spheroids was studied by Elipe and Ferrer [13] and El-Shaboury and El-Tantawy [14]. When one or two of the primaries are triaxial bodies, it was studied by El-Shaboury et al. [15], Khanna and Bhatnagar [16] and Sharma et al. [17].

Some of the illustrious work is done by Danby [18], Sharma and Subba Rao [19]; Subba Rao and Sharma [20] in the perturbed restricted three-body problem. Sharma [21, 22] included the oblateness of the more massive and small primary, respectively, in the photogravitational problem and studied the periodic solutions around the Lagrangian points. Orbital resonance in RTBP was studied by Peale [23], Greenberg [24] and Hadjidemetriou [25]. Further, Dutt and Sharma [26] studied the effect of the solar radiation pressure on the periodic orbits in the Sun-Mars system. Beevi and Sharma [27] considered the oblateness of the more massive primary (Saturn) and studied the periodic and quasi-periodic orbits in the Saturn-Titan system. Furthermore, the existence of liberation points and their linear stability as well as periodic orbits around these points when the more massive primary is radiating and the smaller is an oblate spheroid were studied by Abouelmagd and Sharaf [28]. Singh and Haruna [29] studied the periodic orbits around the triangular equilibrium points when the three participating bodies are oblate spheroids, under the effect of radiation of the main masses and small change in the Coriolis and centrifugal forces. Zotos [30] studied the case of planar restricted three-body problem by considering one of the two primaries as an oblate spheroid to study the escape and crash mechanism of periodic orbits.

The effect of radiation pressure of a source can be expressed by a mass reduction factor $q = 1-\varepsilon$, where the radiation coefficient ε is the ratio of the force F_p which is caused by radiation to the force F_g which results from gravitation, i.e., $\varepsilon = F_p / F_g$. q is expressed in terms of particle radius 'a', density 'δ' and radiation pressure efficiency 'χ' (in CGS system) as

$$q = 1 - \frac{5.6\times 10^{-5}}{a\delta}\chi$$

Knowing the mass and the luminosity of the radiating body, ε can be found for any given radius and density. Solar radiation pressure force F_p changes with distance by the same law of gravitational attraction force F_g and acts opposite to it. Thus, Sun's resulting force acting on the particle is (Sharma [22]; Kalvouridis et al. [31]).

$$F = F_g - F_p = \left(1 - F_p / F_g\right)F_g = \left(q\right)F_g$$

Population of asteroids in the Jovian first-order mean motion resonances 2:1, the Hecuba gap 3:2, the Hilda group 4:3 are closely linked to the orbital evolution of the giant planets. This is because of their orbital proximity to Jupiter (Brož and Vokrouhlicky [32]). In this study, we consider the more massive primary as a source of radiation and the smaller primary as an oblate spheroid. The more massive primary is the Sun and the oblate spheroid is Jupiter. The periodic orbits around the Sun with 4:3, 3:2 and 2:1 first-order interior resonances in the framework of Sun-Jupiter photogravitational restricted three-body problem (PRTBP) are studied using Poincaré surface of section (PSS) method. The study has been carried out to find the change in the period of these orbits with the effects of solar radiation pressure (SRP) and oblateness.

2 Equations of Motion

In the dimensionless synodic coordinate system with origin of the system positioned on the center of mass of the primaries, considering the more massive primary $(-\mu, 0)$ as a source of radiation and smaller primary $(1-\mu, 0)$ as an oblate spheroid with its equatorial plane coincident with the plane of motion, the equations of motion of the third body are (Sharma [22])

$$\ddot{x} - 2n\dot{y} = \frac{\partial \Omega}{\partial x} \tag{1}$$

$$\ddot{y} + 2n\dot{x} = \frac{\partial \Omega}{\partial y} \tag{2}$$

$$\Omega = \frac{n^2}{2}\left[\left(1-\mu\right)r_1^2 + \mu r_2^2\right] + \frac{q(1-\mu)}{r_1} + \frac{\mu}{r_2} + \frac{\mu A_2}{2r_2^3} \tag{3}$$

$$r_1^2 = \left(x+\mu\right)^2 + y^2$$

$$r_2^2 = \left(x+1-\mu\right)^2 + y^2$$

and

$$\mu = m_2 / (m_1 + m_2) \leq \frac{1}{2}$$

m_1 and m_2, $m_1 > m_2$ are the dimensional masses of the primaries.
The mean motion n of the primaries is given by

$$n^2 = 1 + \frac{3}{2} A_2 \text{ with } A_2 = \left(AE^2 - AP^2\right) / 5R^2;$$

where AE and AP are dimensional equatorial and polar radii of the smaller primary and R is the distance between the primaries. r_1 and r_2 are the distances from the more massive and smaller primary, respectively.

The Jacobi integral is

$$\dot{x}^2 + \dot{y}^2 = 2\Omega - C \tag{4}$$

3 Poincaré Surfaces of Section

PSS is a widely used technique in locating the periodic and quasi-periodic orbits. To determine the orbital elements of the test particles at any instant, it is necessary to know its initial position $(x,\ y)$ and velocity $(\dot{x}, \dot{y})$ which corresponds to a point in a four-dimensional phase space. We have constructed the PSS on the $x, \dot{x}$ plane. The initial values were selected along the x-axis by using intervals of length between 0.0001 and 0.01. The magnitude of velocity vector was determined from Jacobi constant C. Fine discretization of positions along the x-axis guarantees an extensive coverage of the phase plane, since each trajectory regardless of the complexity of its motion has a unique path through the phase plane. By defining the plane, say $y = 0$, in resulting three-dimensional space, the values of x and $\dot{x}$ can be plotted every time the particle has $y = 0$, whenever trajectory intersects the plane in a particular direction, say $\dot{y} > 0$.

4 Periodic Orbits around Sun in the Sun-Jupiter System

For the Sun-Jupiter system, we have generated the PSS for the mass ratio of $\mu = 0.0009537284$, taken from Sharma and Subba Rao [33], for the Jacobian constant $C = 2.99$ and $q = 1$ as shown in Figure 1 and for q = 0.99 and 0.985, as shown in Figures 2 and 3, respectively. Fourth-order Runge-Kutta-Gill method is used to integrate the equations of motion (1) and (2) for generating Poincaré surface of sections. For choosing the initial conditions for numerical integration, constant values of solar radiation pressure q is taken with different values of C. The location of periodic orbits with 4:3, 3:2 and 2:1 interior resonances (located from PSS) around the Sun are shown in Figure 4.

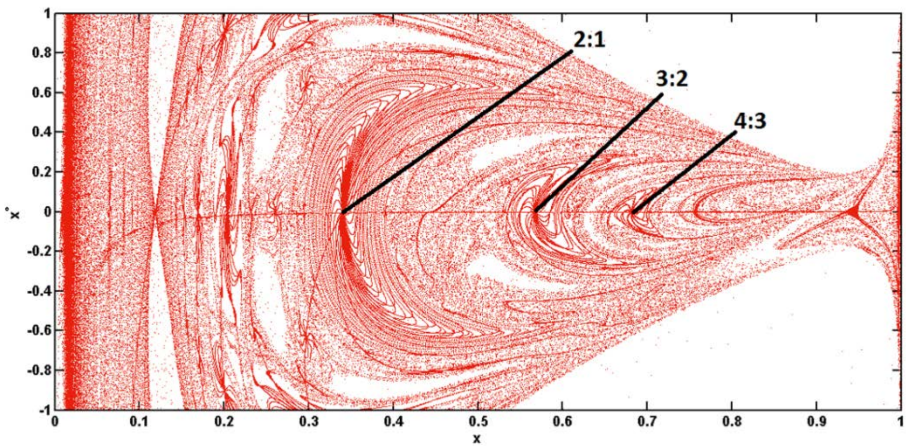

Figure 1. PSS of Sun-Jupiter system for $C = 2.99$ and $q = 1$.

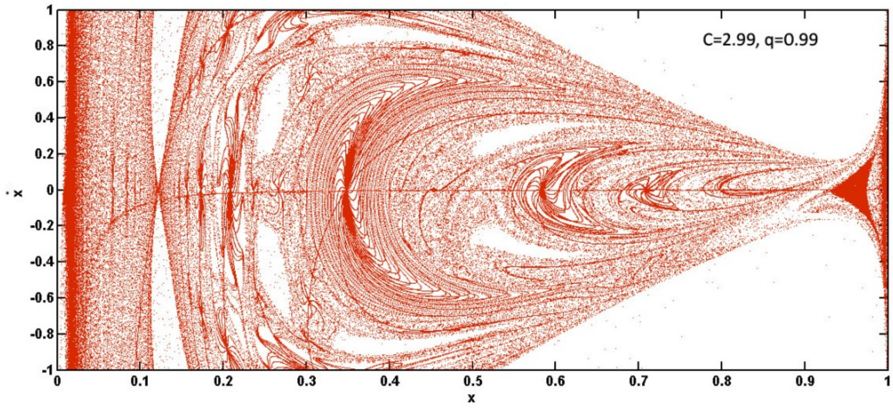

Figure 2. PSS of Sun-Jupiter system for $C = 2.99$ and $q = 0.99$.

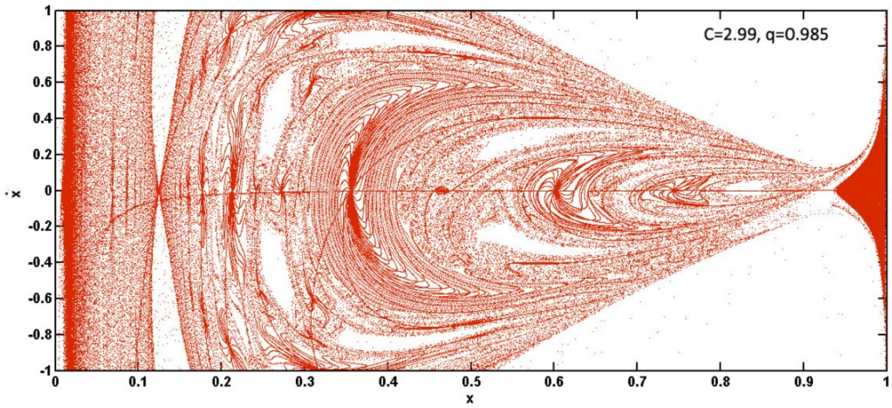

Figure 3. PSS of Sun-Jupiter system for $C = 2.99$ and $q = 0.985$.

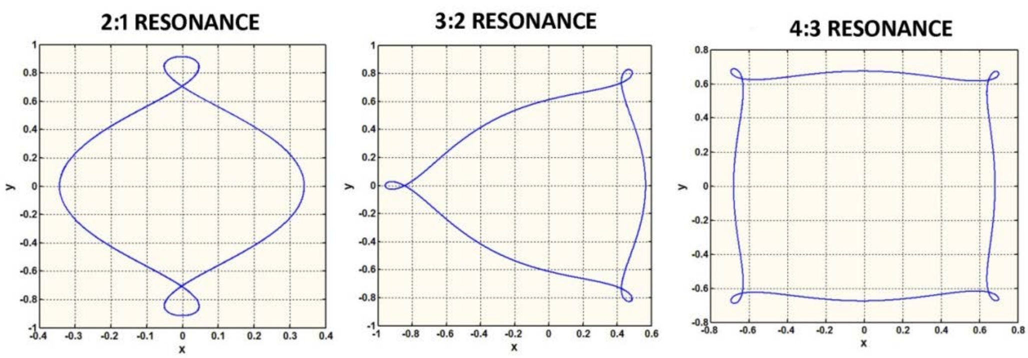

Figure 4. Periodic orbits around the Sun in Sun-Jupiter system for $C = 2.99$ and $q = 1$.

The periodic orbits are located from the PSS of Sun-Jupiter system and are plotted as in Figure 4, showing the interior resonance orbits around the Sun in the Sun-Jupiter system. The resonance of these periodic orbits is shown to become tidal or gravitational lock with resonance 1:1. The transformation of these periodic orbits with interior resonances of 2:1, 3:2 and 4:3 is shown in Figures 5 to 7, respectively, by considering constant values of radiation pressure q and oblateness coefficient A_2 and increasing the

Jacobi constant C from 2.99 to 3.15. Similar transformation of tidal or gravitational lock takes place when the constant values of q and A_2 are different by increasing the value of C.

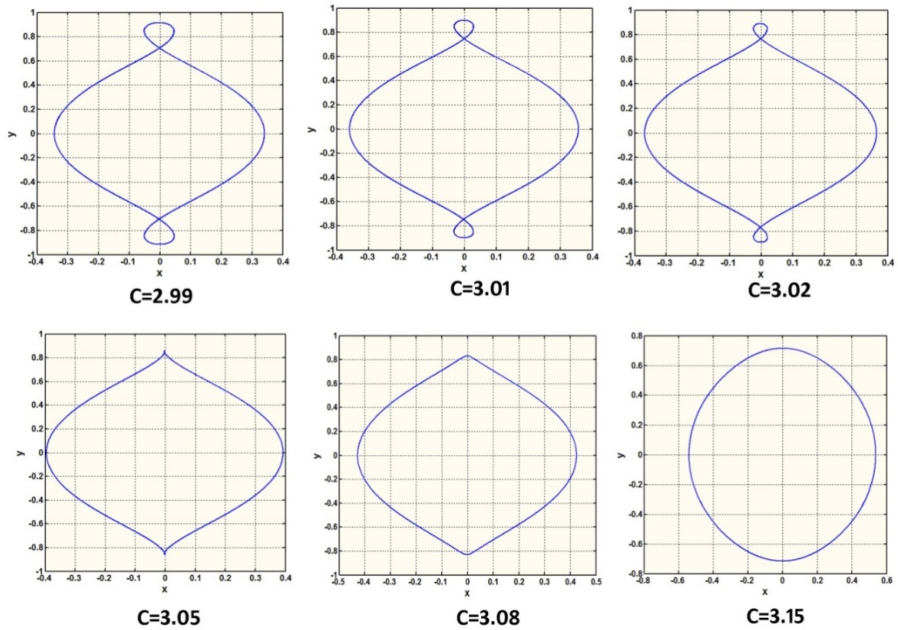

Figure 5. Transformation of 2:1 interior resonance periodic orbit with increase in Jacobi constant for $q = 1$.

It is also observed that the period of time of these orbits increases with the increase in Jacobi constant C for constant values of solar radiation pressure q and oblateness coefficient A_2. We have discussed it in section 6.

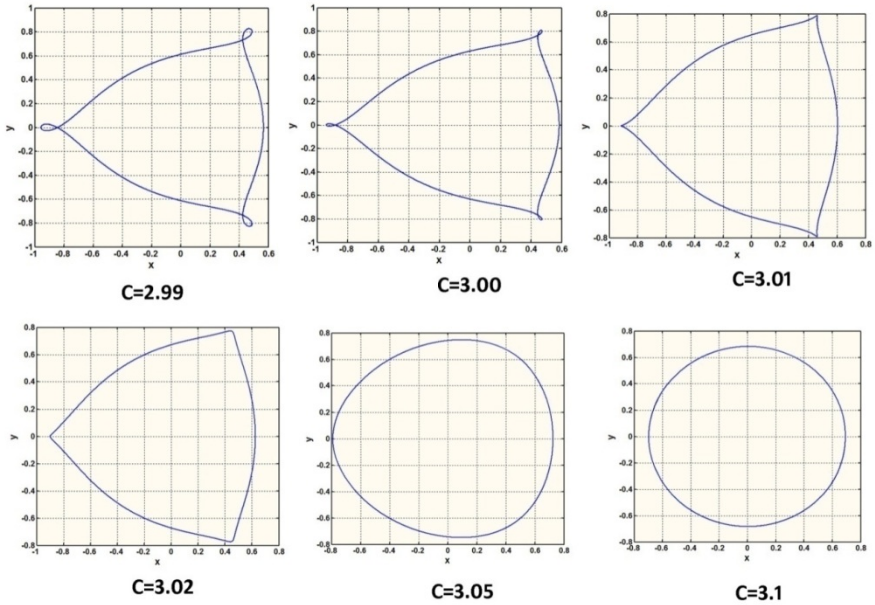

Figure 6. Transformation of 3:2 interior resonance periodic orbit with increase in Jacobi constant for $q = 1$.

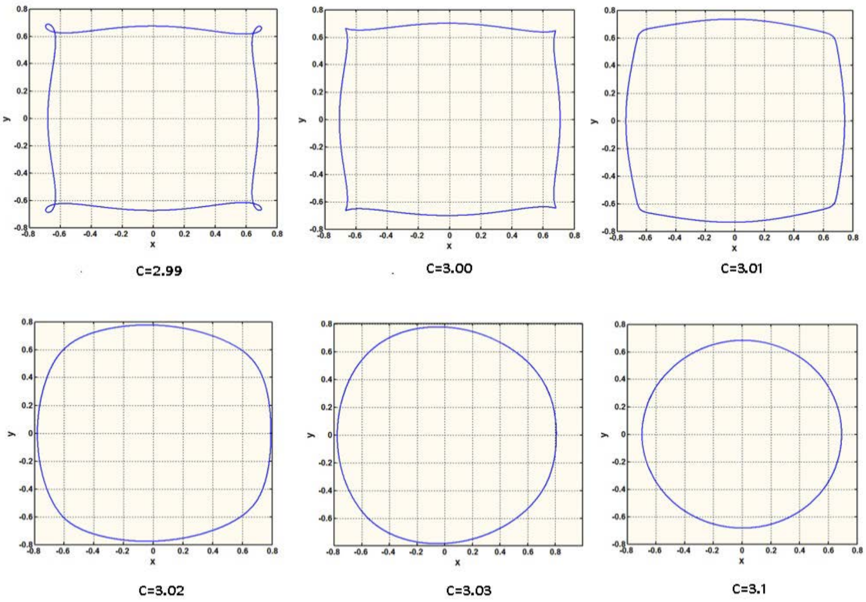

Figure 7. Transformation of 4:3 interior resonance periodic orbit with an increase in Jacobi constant when $q = 1$.

5 Stability of the Periodic Orbits

Kolmogorov-Arnold-Moser (KAM) theory provides the stability condition for the periodic orbits in planar restricted three-body problem. PSS method is used to predict regular and chaotic behavior of a trajectory. For a regular trajectory, there exists a stable region of islands in the surfaces of section or a curve shrinking to a point and a periodic orbit exists there and a closed area around it represents quasi-periodic orbits. Any irregular distribution of points on the surfaces of section that describes the trajectory is chaotic in behavior.

In the present dynamical system considered, the KAM tori of the interior resonance orbits are used to measure the degree of stability of the periodic orbits around the Sun with respect to the region around it in the phase space. Figures 8 to 10 provide the location of the periodic orbit as a function of the Jacobi constant C . These results are generated from the PSS by varying the Jacobi constant C . In Figures 8 to 10, the solid line corresponds to the rightmost tip and the dashed line corresponds to the leftmost tip of the island obtained by PSS of the interior resonance orbits around the Sun.

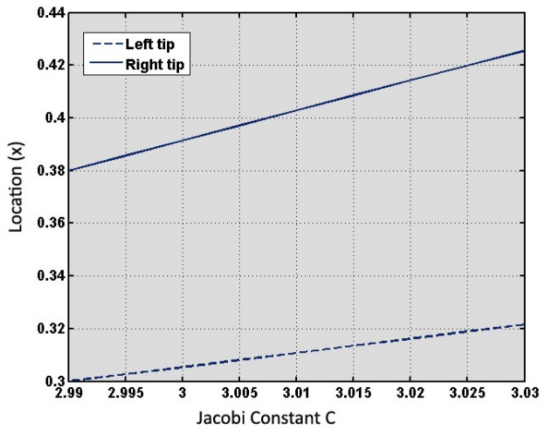

Figure 8. Width of KAM tori and location of 2:1 interior resonance orbit around Sun in Sun-Jupiter system.

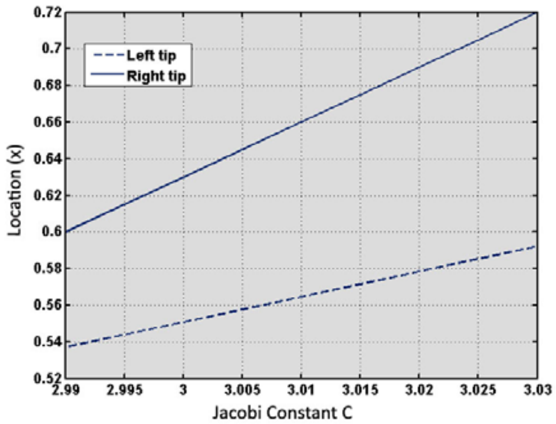

Figure 9. Width of KAM tori and location of 3:2 interior resonance orbits around Sun in Sun-Jupiter system.

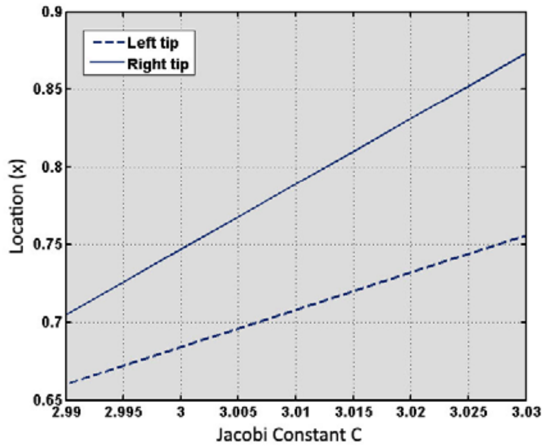

Figure 10. Width of KAM tori and the location of 4:3 interior resonance orbits around Sun in Sun-Jupiter system.

We can notice from Fig. 8 to 10, the width of KAM tori of the first-order interior resonance periodic orbits 2:1, 3:2 and 4:3 with the increase in Jacobi constant C. The location of these periodic orbits moves closer to the more massive primary with the increase in Jacobi constant C. The width of KAM tori is different for the three first-order interior resonance cases. The width of KAM tori for 2:1 resonant orbit is greater than that of 3:2 and 4:3, and it is greater for 3:2 resonant orbit than 4:3 resonant orbit.

6 Effect of Perturbations on Period of Time of the Interior Resonance Periodic Orbits

In the framework of the Photogravitational Restricted Three-Body Problem (PRTBP), the periodic orbits around the Sun in the Sun-Jupiter system with 2:1, 3:2 and 4:3 interior resonances with the orbit of Jupiter are studied. The solar radiation pressure plays an important role in the transformation of resonance of the periodic orbits around the Sun. It also has an effect on the period of time of these interior resonance orbits and shifts the location of these periodic orbits closer to the smaller primary. PSS has been generated for various Jacobi constant C in the Sun-Jupiter system by varying the value of q from 1 to 0.98. Periodic orbits around the Sun are located from the PSS and are plotted. The variation in the period of time is noted and plotted to compare the deviation of the period of time with q. It shows that the period of time increases with the increase in C and decreases with the increase in solar radiation pressure for all the cases of 2:1, 3:2 and 4:3 interior resonance orbits as shown in Figures 11 to 13.

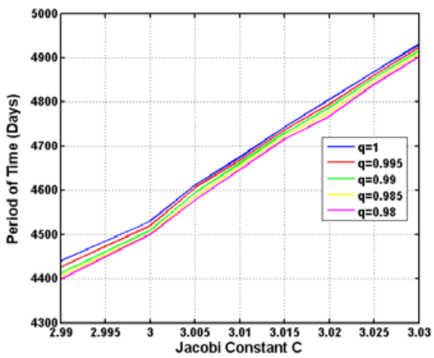

Figure 11. Period of time of 2:1 periodic orbits with solar radiation pressure q and C.

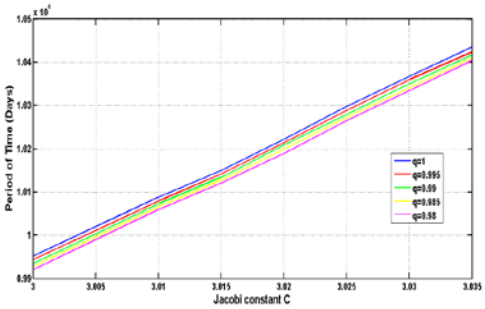

Figure 12. Period of time of 3:2 periodic orbits with solar radiation pressure q and C.

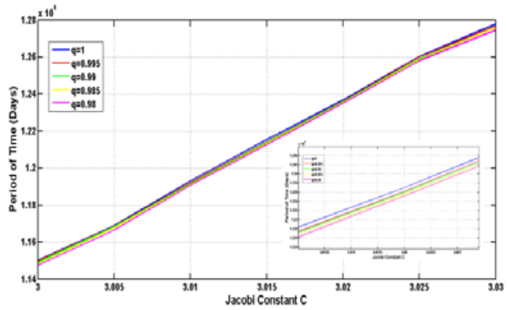

Figure 13. Period of time of 4:3 periodic orbits with solar radiation pressure q and C.

We have considered the effect of oblateness (A_2=2.117 × 10^{-10}) of Jupiter on the motion of the particle in the PRTBP to study the deviation in the location of the periodic orbits around Sun in the Sun-Jupiter system and also the period of time deviations. It is found that there is no significant change in the location of the interior resonance periodic orbits. The time period of these orbits is found to increase marginally with the oblateness of Jupiter. The time period of interior resonance orbits is computed with and without oblateness effect. It is found that the first-order interior resonance orbits with 2:1 resonance have 0.146% increase in the time period. The 3:2 and 4:3 resonance orbits also have increase in the time period by 0.054% and 0.057%, respectively.

7 Summary and Conclusion

The first-order interior resonance orbits 2:1, 3:2 and 4:3 are studied. These orbits transform into nearly circular orbits with resonance 1:1 or tidal lock, a rare case of resonance with 1:1, for Jacobi constant $C \geq 3.1$. For these three type of orbits, the width of KAM tori increases with the increase in Jacobi constant. The location of these periodic orbits moves closer to the more massive primary (Sun) with the increase in Jacobi constant. The width of KAM tori of the periodic orbits 2:1 > 3:2 > 4:3. The time period of these orbits is found to increase with the increase in Jacobi constant. The time period decreases with increase in solar radiation pressure and marginally increases with oblateness of Jupiter.

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