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Note on “An Easy Method to Derive EOQ and EPQ Inventory Models with Backorders”

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Abstract Cárdenas-Barrón (2010) applied algebraic methods to EOQ and EPQ models without referring to differential equations, allowing researchers without backgrounds in calculus to understand inventory models with ease. In this note, we point out that the derivation for EPQ model can be obtained by a transformation of the EOQ model and then we provide a further simplification of his approach such that future practitioners can realize his important findings and apply algebraic methods in their own research.

Keywords: Algebraic method, Cauchy-Bunyakovsky-Schwarz (CBS) inequality, arithmetic-geometric mean (AGM) inequality.

1 Introduction

Grubbstrom and Erdem [1] stated that algebraic methods can be applied as an alternative approach to teach inventory models to practitioners who are not familiar with calculus. There are several papers that have used algebraic methods to solve the optimal solution for inventory systems. In Cárdenas-Barrón [2] a literature review for inventory models with respect to algebraic methods is provided. We point out that Ronald et al. [3], Lan et al. [4] and Lin et al. [5] presented different algebraic approach to solve inventory models. In this note, we provide further discussion of Cárdenas-Barrón [6] that referred to CBS and AGM inequalities to derive the optimal solution for inventory models with linear backorder cost. We will show that the full potential of the solution procedure of Cárdenas-Barrón [6] is not demonstrated in his own paper. Neither did Cárdenas-Barrón [6] conduct a thorough investigation into the connection between the EOQ model and the EPQ models. The lengthy solution approach for EOQ and EPQ models may confuse readers when interpreting the elegant proof proposed by Cárdenas-Barrón [6] for the EOQ model. In the following, a brief overview of recent literature is provided. Currently, there are eleven papers mentioning Cárdenas-Barrón [6] in their References. Widyadana and Wee [4] adopted algebraic approaches to locate the optimal solution for inventory models with product recovery of Teunter [8]. Cárdenas-Barrón [9] developed a vendor-buyer integrated inventory system without shortages. Teng et al. [10] revised a typo in the integrated vendor-buyer system of Wee and Chung [11] and then derived the corrected optimal solution by algebraic methods. Widyadana et al. [12] applied a simplified model to approximate an inventory system with deteriorated items. Zhang et al. [13] studied a multi-item inventory model by joint replenishment approach and then developed a mixed integer non-linear programming algorithm and used heuristic methods to find the optimal solution. Chung [14] considered inventory models with partially permissible delay in payments and solved the optimal problem by using analytical methods. Gambini et al. [15] used analytical methods to find mathematical properties of EOQ models. Andriolo et al. [16] is a review paper containing 219 articles till present, creating a comprehensive survey for the past one hundred years of EOQ model research. San Jose et al. [17] developed an EPQ model with partial backorder and lost sales to consider the mixture of (a) last in, first out, and (b) first in, first out policies. We can conclude that the above nine papers only mentioned Cárdenas-Barrón [6] in their Introductions. The course of research in the above mentioned inventory models papers veer in a very different direction than that of Cárdenas-Barrón [6]. There are two related articles: Cárdenas-Barrón [2] and Sphicas [18] for Cárdenas-Barrón [6]. The first article, Cárdenas-

Barrón [2] considered EOQ models with two backorder costs: linear and fixed such that Cárdenas-Barrón [2] is an extension of Cárdenas-Barrón [6]. When the production cost and the fixed backorder cost are neglected, Cárdenas-Barrón [2] will revert to Cárdenas-Barrón [6]. The second article, Sphicas [18] extended EOQ models with linear and fixed backorder costs to the coefficient of backorder attractiveness. However, in Cárdenas-Barrón [2] and Sphicas [18], both of these articles did not give any further discussion of Cárdenas-Barrón [6].

2 Notation and Assumptions

To be compatible with Cárdenas-Barrón [6], we use the same notation as his except two new expressions to convert an EPQ model to an EOQ model.

d = demand rate per time unit,
A = ordering cost per order,
h = per unit holding cost per unit time,
v = per unit backorder cost per unit time,
p = production rate per unit time,
Q = order quantity,
B = backorders level.
The two new expressions proposed by us:

$A_0 = A\left(1 - d/p\right)$,

$Q_0 = Q\left(1 - d/p\right)$.

3 Review of Cárdenas-Barrón [6]

For an EOQ model with backorders, we recall the excellent approach proposed by Cárdenas-Barrón [6]. The total inventory cost, TC(Q,B), is denoted as

$$TC\left(Q,B\right) = \frac{Ad}{Q} + \frac{h\left(Q-B\right)^2}{2Q} + \frac{vB^2}{2Q} \tag{3.1}$$

In the following, we provide an outline of his sophistic derivations. For reference of the detailed derivations, please refer to Equations (2-11) of Cárdenas-Barrón [6]. Cárdenas-Barrón [6] applied the CBS inequality to show that

$$TC\left(Q,B\right) \geq \frac{Ad}{Q} + \frac{Q}{2}\left\{\left(\frac{\sqrt{hv}}{\sqrt{h+v}}\right)\left(1-\frac{B}{Q}\right) + \left(\frac{\sqrt{hv}}{\sqrt{h+v}}\right)\left(\frac{B}{Q}\right)\right\}^2 \tag{3.2}$$

the minimum of which occurs at

$$\frac{\sqrt{h}}{\sqrt{v}}\left(1-\frac{B}{Q}\right) = \frac{\sqrt{v}}{\sqrt{h}}\frac{B}{Q} \tag{3.3}$$

to find that

$$hQ^* = \left(h+v\right)B^* . \tag{3.4}$$

Cárdenas-Barrón [6] simplified Equation (3.2) as

$$TC\left(Q,B\right) \geq \frac{Ad}{Q} + \frac{hvQ}{2\left(h+v\right)} \tag{3.5}$$

to apply that

$$\frac{Ad}{Q^*} = \frac{hvQ^*}{2\left(h+v\right)} \tag{3.6}$$

and

$$TC\left(Q^*,B^*\right)=\sqrt{\frac{2Adhv}{h+v}} \tag{3.7}$$

For an EPQ model, the total cost is expressed as

$$TC\left(Q,B\right)=\frac{Ad}{Q}+\frac{h\left(Q\left(1-d/p\right)-B\right)^2}{2Q\left(1-d/p\right)}+\frac{vB^2}{2Q\left(1-d/p\right)} \tag{3.8}$$

In Cárdenas-Barrón [6], the approach was repeated to derive similar results in Equations (13-22) of his paper. The purpose of this note is to provide a simplification of his solution procedure for the EPQ model.

4 Our Simplification

We adopt two new notations: $A_0=A\left(1-d/p\right)$ and $Q_0=Q\left(1-d/p\right)$, then we convert Equation (3.8) as

$$TC\left(Q,B\right)=\frac{A_0 d}{Q_0}+\frac{h\left(Q_0-B\right)^2}{2Q_0}+\frac{vB^2}{2Q_0}. \tag{4.1}$$

If we overlook the subscript, then Equation (4.1) is identical to Equation (3.1). Hence, we find the minimum point

$$hQ_0^{\;*}=\left(h+v\right)B^*, \tag{4.2}$$

$$\frac{A_0 d}{Q_0^{\;*}}=\frac{hvQ_0^{\;*}}{2\left(h+v\right)}, \tag{4.3}$$

and

$$TC\left(Q_0^{\;*},B^*\right)=\sqrt{\frac{2A_0 dhv}{h+v}}. \tag{4.4}$$

Our findings of Equations (4.2-4.4) are the same results as the final findings of Equations (20-22) in Cárdenas-Barrón [6], without his repeated derivations, thus illustrating that our simplification is valid.

5 Conclusion

We apply two new expressions to find the relation between EOQ and EPQ models such that the derivation of the EOQ model proposed by Cárdenas-Barrón [6] can be directly used for the solution approach of the EPQ model. Consequently, his tedious solution approach for the EPQ model can be removed. Our findings will help readers easily understand the power of Cárdenas-Barrón's algebraic procedure for EOQ models.

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